Advt. No. 28/2022 Dated 7/12/2022

Exam held on - 12.02.2024

MATH/ARO/II/24

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Test Booklet No. : 00009

Series A

TEST BOOKLET

MATHEMATICS

Time Allowed : 2 Hours **Full Marks : 100**

Read the following instructions carefully before you begin to answer the questions :

1. The name of the Subject, Roll Number as mentioned in the Admission Certificate, Test Booklet No. and Series are to be written legibly and correctly in the space provided on the Answer-Sheet with Black/Blue ballpoint pen.
2. **Answer-Sheet without marking Series as mentioned above in the space provided for in the Answer-Sheet shall not be evaluated.**
3. All questions carry equal marks.
   The Answer-Sheet should be submitted to the Invigilator.
   Directions for giving the answers : Directions for answering questions have already been issued to the respective candidates in the 'Instructions for marking in the OMR Answer-Sheet' along with the Admit Card and Specimen Copy of the OMR Answer-Sheet.
   Example :
   Suppose the following question is asked :
   The capital of Bangladesh is
   (A) Chennai
   (B) London
   (C) Dhaka
   (D) Dhubri
   You will have four alternatives in the Answer-Sheet for your response corresponding to each question of the Test Booklet as below :
   Ⓐ Ⓑ Ⓒ Ⓓ
   In the above illustration, if your chosen response is alternative (C), i.e., Dhaka, then the same should be marked on the Answer-Sheet by blackening the relevant circle with a Black/Blue ballpoint pen only as below :
   Ⓐ Ⓑ ● Ⓓ
   The example shown above is the only correct method of answering.
4. Use of eraser, blade, chemical whitener fluid to rectify any response is prohibited.
5. Please ensure that the Test Booklet has the required number of pages (20) and 100 questions immediately after opening the Booklet. In case of any discrepancy, please report the same to the Invigilator.
6. No candidate shall be admitted to the Examination Hall/Room 20 minutes after the commencement of the examination.
7. No candidate shall leave the Examination Hall/Room without prior permission of the Supervisor/Invigilator. No candidate shall be permitted to hand over his/her Answer-Sheet and leave the Examination Hall/Room before expiry of the full time allotted for each paper.
8. No Mobile Phone, Electronic Communication Device, etc., are allowed to be carried inside the Examination Hall/Room by the candidates. Any Mobile Phone, Electronic Communication Device, etc., found in possession of the candidate inside the Examination Hall/Room, even if on off mode, shall be liable for confiscation.
9. No candidate shall have in his/her possession inside the Examination Hall/Room any book, notebook or loose paper, except his/her Admission Certificate and other connected papers permitted by the Commission.
10. Complete silence must be observed in the Examination Hall/Room. No candidate shall copy from the paper of any other candidate, or permit his/her own paper to be copied, or give, or attempt to give, or obtain, or attempt to obtain irregular assistance of any kind.
11. This Test Booklet can be carried with you after answering the questions in the prescribed Answer-Sheet.
12. Noncompliance with any of the above instructions will render a candidate liable to penalty as may be deemed fit.
13. No rough work is to be done on the OMR Answer-Sheet. You can do the rough work on the space provided in the Test Booklet.

N.B. : There will be negative marking @ 0·25 per 1 (one) mark against each wrong answer.

SEAL

 [No. of Questions : 100]

1. For what value of k does the equation

$$12x^2 + 7xy + ky^2 + 13x - y + 3 = 0$$

represent two straight lines?

(A) 10

(B) −10

(C) 20

(D) −20

2. The equation of the circle which touches the line $5x + 12y = 1$ and which has its centre at (3, 4) is

(A) $(x-3)^2 + (y-4)^2 = \left(\frac{62}{13}\right)^2$

(B) $(x-3)^2 + (y-4)^2 = 2^2$

(C) $(x-3)^2 + (y-4)^2 = \left(\frac{61}{12}\right)^2$

(D) $(x-4)^2 + (y-3)^2 = \left(\frac{62}{13}\right)^2$

3. Prove that the circles

$$x^2 + y^2 + 2ax + c^2 = 0$$

and

$$x^2 + y^2 + 2by + c^2 = 0$$

touch each other if

(A) $a^2 + b^2 = c^2$

(B) $a^2 + b^2 + c^2 = 0$

(C) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

(D) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = 0$

4. The equation of the parabola, whose focus is $\left(\frac{5}{4}, 1\right)$ and directrix is $4x - 13 = 0$, is

(A) $y^2 - 2y - 4x - 8 = 0$

(B) $y^2 + 2y + 4x - 8 = 0$

(C) $y^2 - 2y + 4x + 8 = 0$

(D) $y^2 - 2y + 4x - 8 = 0$

5. The condition that the line $lx + my + n = 0$ may touch the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

is

(A) $a^2l^2 + b^2m^2 = n^2$

(B) $\frac{a^2}{l^2} + \frac{b^2}{m^2} = n^2$

(C) $a^2l^2 + b^2m^2 + n^2 = 0$

(D) $(a^2l^2 + b^2m^2)n^2 = 1$

6. The equation of the sphere passing through the origin and the points $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ is

(A) $x^2 + y^2 + z^2 + ax + by + cz = 0$

(B) $x^2 + y^2 + z^2 - ax - by - cz = 0$

(C) $x^2 + y^2 + z^2 + ax - by - cz = 0$

(D) $x^2 + y^2 + z^2 - ax + by - cz = 0$

7. The vector equation for the line passing through the points (–1, 0, 2) and (3, 4, 6) is

(A) $\vec{r} = -\hat{i} + 2\hat{k} + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$

(B) $\vec{r} = -\hat{i} + \lambda(4\hat{j} + 4\hat{k})$

(C) $\vec{r} = 2\hat{k} + \lambda(4\hat{i} + 4\hat{j})$

(D) None of the above

8. Two systems of rectangular axes have the same origin. If a plane cuts them at distances a, b, c and p, q, r from the origin, then

(A) $a + b + c = p + q + r$

(B) $a^{-1} + b^{-1} + c^{-1} = p^{-1} + q^{-1} + r^{-1}$

(C) $a^{-2} + b^{-2} + c^{-2} = p^{-2} + q^{-2} + r^{-2}$

(D) $a = p$, $b = q$ and $c = r$

9. If

$$\lambda x^2 + 5y^2 + \mu z^2 - axy + 2\lambda x - 4\mu y - 2az + d = 0$$

represents a sphere of radius 2, then the value of d is

(A) 120

(B) 121

(C) 123

(D) 139

10. If α and β are the roots of $x^2 - 2x + 4 = 0$, then the value of $\alpha^n + \beta^n$ is

(A) $2^{n+1} \cos\left(\frac{n\pi}{3}\right)$

(B) $2^{n+1} \sin\left(\frac{n\pi}{3}\right)$

(C) $2^n \cos\left(\frac{n\pi}{3}\right)$

(D) $2^{\frac{n+1}{2}} \cos\left(\frac{n\pi}{3}\right)$

11. Suppose $ax^3 + bx^2 + cx + d = 0$ has three roots α, β, β. Then choose the correct answer.

(A) $\alpha + 2\beta = -\frac{b}{a}$, $\alpha\beta^2 = \frac{d}{a}$

(B) $\alpha + 2\beta = -\frac{b}{a}$, $\alpha\beta^2 = -\frac{d}{a}$

(C) $\alpha + 2\beta = \frac{b}{a}$, $\alpha\beta^2 = \frac{d}{a}$

(D) None of the above

Consider $a \neq 0$.

12. The value of

$$\lim_{n \to \infty} \frac{[(n+1)(n+2)\cdots 2n]^{\frac{1}{n}}}{n}$$

is

(A) $\frac{4}{e}$

(B) $\frac{e}{4}$

(C) e

(D) 4

13. The relative maximum of $f(x) = x^2 e^x$ is

(A) $\frac{4}{e^2}$

(B) $\frac{4}{e}$

(C) $\frac{1}{e^2}$

(D) e^2

14. The value of $\int_1^2 \frac{dx}{(x^2 - 2x + 4)^{\frac{3}{2}}}$ is

(A) $\frac{1}{3}$

(B) $\frac{1}{4}$

(C) $\frac{1}{5}$

(D) $\frac{1}{6}$

15. Given

$$\phi(\alpha) = \int_{\alpha}^{\alpha^2} \frac{\sin(\alpha x)}{x} dx, \quad \alpha \neq 0$$

Then the value of $\frac{d\phi}{d\alpha}$ is

(A) $\frac{3\sin\alpha^3}{\alpha}$

(B) $\frac{2\sin\alpha^2}{\alpha}$

(C) $\frac{3\sin\alpha^3 - 2\sin\alpha^2}{\alpha}$

(D) None of the above

16. If

$$u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$$

then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

(A) $\sin 2u$

(B) $\cos 2u$

(C) $\tan u$

(D) $\tan 2u$

17. The nth derivative of

$$y = \frac{x^2}{(x+2)(2x+3)}$$

is

(A) $\frac{(-1)^n n!}{2}\left[\frac{9 \cdot 2^n}{(2x+3)^{n+1}} - \frac{8}{(x+2)^{n+1}}\right]$

(B) $\frac{(-1)^n \cdot n!}{2}\left[\frac{9 \cdot 2^n}{(2x+3)^{n+1}} + \frac{8}{(x+2)^{n+1}}\right]$

(C) $\frac{(-1)^n n!}{2}\left[\frac{9 \cdot 2^n}{(2x+3)^n} + \frac{8}{(x+2)^{n+1}}\right]$

(D) $\frac{(-1)^n \cdot n!}{2}\left[\frac{9 \cdot 2^n}{(2x+3)^{n+1}} + \frac{8}{(x+2)^n}\right]$

18. The points of inflexion on the curve $y = (\log x)^3$ are

(A) (0, 1) and (2, 8)

(B) (1, 0) and (2, 8)

(C) (0, 1) and (e, 8)

(D) (1, 0) and (e^2, 8)

19. The area of the cardioid $r = 2a(1+\cos\theta)$ is

(A) $6\pi a^2$

(B) $5\pi a^2$

(C) $4\pi a^2$

(D) $3\pi a^2$

20. If n is even, then $\int_0^{\frac{\pi}{2}} \sin^n x\,dx$ is

(A) $\left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)\cdots\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}$

(B) $\left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)\cdots\frac{4}{5}\cdot\frac{2}{3}\cdot 1$

(C) $\left(\frac{n-2}{n-1}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)\cdots\frac{3}{4}\cdot\frac{1}{2}\cdot\pi$

(D) $\left(\frac{n-2}{n-1}\right)\left(\frac{n-3}{n-4}\right)\left(\frac{n-5}{n-4}\right)\cdots\frac{4}{5}\cdot\frac{2}{3}\cdot 1$

21. If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x\,dx$, then choose the correct answer.

(A) $I_n + I_{n-1} = \frac{1}{n-1}$

(B) $I_n + I_{n-1} = \frac{1}{n+1}$

(C) $I_n + I_{n-2} = \frac{1}{n-1}$

(D) $I_n + I_{n-2} = \frac{1}{n+1}$

22. If two vectors $\vec{a} = 5\hat{i} + \hat{j} - 7\hat{k}$ and $\vec{b} = \lambda\hat{i} + \hat{j} - 7\hat{k}$ are parallel, then the value of λ is

(A) 1

(B) 7

(C) 2

(D) 5

23. If

$\vec{f} = \cos xy\,\hat{i} + (3xy - 2x^2)\hat{j} - (3x+2y)\hat{k}$

then $\frac{\partial^2 \vec{f}}{\partial x\,\partial y}$ is equal to

(A) $(xy\cos xy + \sin xy)\hat{i} + 3\hat{j}$

(B) $(xy\sin xy + \cos xy)\hat{j} - \hat{k}$

(C) $-(xy\cos xy + \sin xy)\hat{i} + 3\hat{j}$

(D) $-xy\cos xy\,\hat{i} + \hat{k}$

24. If $\vec{r} = t\hat{i} - t^2\hat{j} + (t-1)\hat{k}$ and $\vec{s} = 2t^2\hat{i} + 6t\hat{k}$, then $\int_0^2 \vec{r}\times\vec{s}\,dt$ is

(A) $24\hat{i} + \left(\frac{40}{3}\right)\hat{j} + \left(\frac{64}{5}\right)\hat{k}$

(B) $-24\hat{i} + \left(\frac{40}{3}\right)\hat{j} - \left(\frac{64}{5}\right)\hat{k}$

(C) $-24\hat{i} - \left(\frac{40}{3}\right)\hat{j} + \left(\frac{64}{5}\right)\hat{k}$

(D) $-24\hat{i} - \left(\frac{40}{3}\right)\hat{j} - \left(\frac{64}{5}\right)\hat{k}$

25. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is a vector, then curl $\vec{r}$ is

(A) $-\hat{i}$

(B) $\vec{0}$

(C) $\hat{j}$

(D) None of the above

26. If $f(x, y, z) = 3x^2y - x^2 + zy$, then grad f at the point $(1, -2, -1)$ is

(A) $-14\hat{i} - 2\hat{j} + 2\hat{k}$

(B) $-14\hat{i} + 2\hat{j} - 2\hat{k}$

(C) $7\hat{i} + \hat{j} + 6\hat{k}$

(D) $5\hat{i} - \hat{j} + \hat{k}$

27. If $\vec{A}$ and $\vec{B}$ are irrotational, then

(A) curl $\vec{A} = 0$, curl $\vec{B} = 0$ and $\vec{A} \times \vec{B}$ is not solenoidal

(B) div $\vec{A} = 0$, div $\vec{B} = 0$ and $\vec{A} \times \vec{B}$ is not solenoidal

(C) curl $\vec{A} = 0$, curl $\vec{B} = 0$ and $\vec{A} \times \vec{B}$ is solenoidal

(D) div $\vec{A} = 0$, div $\vec{B} = 0$ and $\vec{A} \times \vec{B}$ is solenoidal

28. The volume of the parallelepiped whose edges are $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$ is

(A) –5

(B) 5

(C) –7

(D) 7

29. An integer is chosen at random from $1, 2, ..., 199, 200$. The probability that the integer is divisible by 8 or 6 is

(A) $\frac{1}{8}$

(B) $\frac{1}{4}$

(C) $\frac{1}{16}$

(D) $\frac{1}{2}$

30. A, B and C are three mutually exclusive and exhaustive events associated with a random experiment. If $P(B) = \frac{3}{2}P(A)$ and $P(C) = \frac{1}{2}P(B)$, then $P(A)$ is equal to

(A) $\frac{4}{13}$

(B) $\frac{2}{13}$

(C) $\frac{1}{13}$

(D) None of the above

31. A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the box at random at a time. The probability that among the balls drawn, there is at least one ball of each colour is approximately

(A) 0·6

(B) 0·7

(C) 0·5

(D) None of the above

32. Let the random variable X have the distribution $P(X=0)=P(X=2)=p$; $P(X=1)=1-2p$; for $0 \le p \le \frac{1}{2}$. For what value of p is the $\text{var}\, X$ a maximum?

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) None of the above

33. In the Poisson distribution, the values of mean and variance are

(A) 0 and 1 respectively

(B) 1 and 0 respectively

(C) equal

(D) None of the above

34. If a and b are constants, then choose the correct answer.

(A) $\text{cov}(X+a, Y+b) = \text{cov}(X, Y)$

(B) $\text{cov}(X, bY) = \text{cov}(X, Y)$

(C) $\text{var}(aX+b) = a^2\, \text{var}(X) + b^2$

(D) $\text{cov}(aX, bY) = \text{cov}(X, Y)$

X, Y are random variables.

35. A random variable X has the following probability function :

$X=x$	:	-2	-1	0	1	2	3
$P(x)$	:	0·1	k	0·2	$2k$	0·3	$3k$

Then the value of $P(X<2)$ is

(A) 0·7

(B) 0·5

(C) 0·3

(D) 0·15

36. In the case of binomial distribution, the recurrence relation for probabilities is

(A) $f(x+1) = \frac{n}{x+1}\frac{p}{q}f(x)$

(B) $f(x+1) = \frac{n}{x-1}\frac{p}{q}f(x)$

(C) $f(x+1) = \frac{n-x}{x+1}\frac{p}{q}f(x)$

(D) $f(x+1) = \frac{n-x}{x-1}\frac{p}{q}f(x)$

Consider $f(x) = b(x; n, p)$ with its usual meaning.

37. The binomial distribution having parameters n, p and $q = 1 - p$ yields Poisson distribution under which of the following conditions?

(A) $p(\text{or } q) \to 0$, $n \to \infty$ and np is not a finite constant

(B) $p(\text{or } q) \to \infty$, $n \to 0$ and np is not a finite constant

(C) $p \to \infty$, $n \to \infty$ and np is a finite constant

(D) $p(\text{or } q) \to 0$, $n \to \infty$ and np is a finite constant

38. The value of the determinant

$$\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$$

is

(A) -1

(B) 1

(C) 2

(D) 0

39. If A is a non-singular matrix, then

(A) $\text{adj}(\text{adj}\, A) = |A|^{n-2} A$

(B) $\text{adj}(\text{adj}\, A) = |A|^{n-1} A$

(C) $\text{adj}(\text{adj}\, A) = |A|^{n} A$

(D) $\text{adj}(\text{adj}\, A) = |A|^{n+1} A$

40. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$$

be a matrix. Then rank A is

(A) 3

(B) 2

(C) 1

(D) 0

41. The minimal polynomial of

$$A = \begin{bmatrix} 4 & -2 & 2 \\ 6 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$$

is

(A) $t^2 + 3t + 2$

(B) $t^2 - 3t + 2$

(C) $t^2 - 3t - 2$

(D) None of the above

42. The characteristic roots of a skew-Hermitian matrix are

(A) either zero or pure imaginary

(B) all real

(C) both pure imaginary and real

(D) of unit modulus

43. The degree of the ordinary differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^4\right]^{\frac{1}{3}}=\frac{d^2y}{dx^2}$$

is

(A) 1

(B) 4

(C) 3

(D) None of the above

44. The solution of the equation

$$(2xy+e^x)y\,dx-e^x dy=0$$

is

(A) $\frac{e^x}{y}+x^2=c$

(B) $e^x y+x^2=c$

(C) $e^x+y=c$

(D) $e^x=c$

where c is an arbitrary constant.

45. The first-order ordinary differential equation whose solution is $(x-c)^2+y^2=4$, c being a constant, is

(A) $\left(\frac{dy}{dx}+1\right)y=4$

(B) $\left(\frac{dy}{dx}+1\right)y^2=4$

(C) $\left\{\left(\frac{dy}{dx}\right)^2+1\right\}y^2=4$

(D) None of the above

46. The solution of the differential equation $x\frac{dy}{dx}+2y=x^2\log x$ is

(A) $y=cx^{-2}+\frac{1}{4}x^2\left(\log x-\frac{1}{4}\right)$

(B) $y=cx^{-2}+\frac{1}{4}x^2\left(\log x+\frac{1}{4}\right)$

(C) $y=cx^2+\frac{1}{4}x^{-2}\left(\log x-\frac{1}{4}\right)$

(D) $y=cx^{-2}+\frac{1}{4}x^{-2}\log x$

47. The differential equation

$$(x^3-2y^2)dx+2xy\,dy=0$$

(A) is exact

(B) is not exact with no integrating factor

(C) is homogeneous

(D) is not exact but has integrating factor

48. The particular integral of the differential equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=\sin 2x$$

is

(A) $-\frac{1}{13}(2\sin 2x+3\cos 2x)$

(B) $-\frac{1}{13}(2\cos 2x+3\sin 2x)$

(C) $-\frac{1}{13}(3\sin 2x-2\sin x)$

(D) $-\frac{1}{13}(2\cos 2x-3\sin 2x)$

49. The radius of convergence of the series

$$\frac{x}{2}+\frac{1\cdot 3}{2\cdot 5}x^2+\frac{1\cdot 3\cdot 5}{2\cdot 5\cdot 8}x^3+\cdots$$

is

(A) $\frac{3}{2}$

(B) $\frac{2}{3}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

50. The series

$$\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^p}$$

converges for

(A) $p \le 1$

(B) $p = 1$

(C) $p > 1$

(D) $p < 1$

51. The alternating series

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$$

is

(A) neither convergent nor absolutely convergent

(B) both convergent and absolutely convergent

(C) convergent but not absolutely convergent

(D) absolutely convergent but not convergent

52. The series $1+\frac{1}{3}+\frac{1}{5}+\cdots+\frac{1}{2n-1}+\cdots$ is

(A) convergent

(B) not convergent

(C) an alternating series

(D) None of the above

53. A function $f:\mathbb{R}\to\mathbb{R}$, $\mathbb{R}=(-\infty, \infty)$ is such that $f^{-1}(G)$ is open in $\mathbb{R}$ whenever G is open in $\mathbb{R}$. Then f is

(A) bounded

(B) unbounded

(C) not continuous

(D) continuous

54. Let $f:\mathbb{R}\to\mathbb{R}$, $\mathbb{R}=(-\infty, \infty)$ be defined by

$$f(x)=\begin{cases} x^2\sin\frac{1}{x^2}, & x\ne 0 \\ 0, & x=0 \end{cases}$$

Then $\frac{d}{dx}f(x)$ is

(A) continuous and unbounded

(B) continuous and bounded

(C) not continuous

(D) None of the above

55. The improper integral

$$\int_0^\infty \frac{e^{-ax}\sin x}{x}\,dx$$

is convergent if

(A) $a < 0$

(B) $a \geq 0$

(C) $a = -1$

(D) None of the above

56. If $f : \mathbb{R} \to \mathbb{R}$ satisfies the relation $f(x+y) = f(x) + f(y)$, $\forall\, x, y \in \mathbb{R}$ and if f is continuous at $x = 0$, then

(A) f is continuous at every point $c \in \mathbb{R}$

(B) f must be monotonically increasing

(C) f must be monotonically decreasing

(D) f is a constant function

57. Consider the sequence $\{x_n\}$, given by $x_n = -n^2$, $n \in \mathbb{N}$. Then the limit inferior of $\{x_n\}$ is

(A) -1

(B) 0

(C) n

(D) None of the above

58. Choose the correct statement.

(A) The unit interval [0, 1] is countable.

(B) The set of all rational numbers in [0, 1] is countable.

(C) The set of irrational numbers in [0, 1] is countable.

(D) None of the above

59. If the interval of differencing is h, then the value of $\Delta(x + \cos x)$ is

(A) $2h\sin\left(x + \frac{h}{2}\right)\sin\frac{h}{2}$

(B) $2h\sin\left(x + \frac{h}{2}\right)\cos\frac{h}{2}$

(C) $h + 2\sin\left(x + \frac{h}{2}\right)\sin\frac{h}{2}$

(D) $h - 2\sin\left(x + \frac{h}{2}\right)\sin\frac{h}{2}$

60. If $f(z)$ and $\overline{f(z)}$ are analytic in a region D, then $f(z) = u(x, y) + iv(x, y)$, $z = x + iy$ is

(A) constant

(B) zero

(C) not continuous

(D) None of the above

61. The bilinear transformation which maps the points $z_1 = 0$, $z_2 = -i$, $z_3 = -1$ into $w_1 = i$, $w_2 = 1$, $w_3 = 0$, respectively is

(A) $w = i\left(\dfrac{z-1}{z+1}\right)$

(B) $w = i\left(\dfrac{z+1}{z-1}\right)$

(C) $w = -i\left(\dfrac{z+1}{z}\right)$

(D) $w = -i\left(\dfrac{z+1}{z-1}\right)$

62. The value of $\int_C \dfrac{z+2}{z}\,dz$, where C is the semi-circle $z = 2e^{i\theta}$, where $0 \le \theta \le \pi$, is

(A) $4 + 2i$

(B) $4 + 2\pi i$

(C) $-4 + 2\pi i$

(D) $-4 - 2i$

63. Let $f(z)$ be analytic inside and on the circle C with centre z_0 and radius r, and let M denote the maximum of $|f(z)|$ on C. Then

(A) $|f^{(n)}(z_0)| \le \dfrac{n!}{r^n}$

(B) $|f^{(n)}(z_0)| \le n!M$

(C) $|f^{(n)}(z_0)| \le \dfrac{Mn!}{r^n}$

(D) None of the above

64. Laurent's series of $\dfrac{1}{z^2 - 3z + 2}$ in $1 < |z| < 2$ is

(A) $\sum_{n=0}^{\infty} \dfrac{z^n}{2^{n+1}}$

(B) $-\sum_{n=0}^{\infty} \dfrac{z^n}{2^{n+1}} - \sum_{n=0}^{\infty} \dfrac{1}{z^{n+1}}$

(C) $-\sum_{n=0}^{\infty} \dfrac{1}{z^{n+1}}$

(D) None of the above

65. The linear transformation $w = iz + i$, $z = x + iy$, maps the right half-plane $\text{Re}(z) \ge 1$ onto

(A) the upper half-plane $\text{Im}(w) \ge 2$

(B) the upper half-plane $\text{Im}(w) \ge 2^{-1}$

(C) the lower half-plane $\text{Im}(w) \le 2$

(D) the lower half-plane $\text{Im}(w) \le 2^{-1}$

66. Let z_0 denote a fixed complex value. If C is a simple closed contour with positive orientation such that z_0 lies interior to C, then

(A) $\int_C \dfrac{dz}{(z - z_0)^n} = 2\pi i$

(B) $\int_C \dfrac{dz}{(z - z_0)^n} = 0$

(C) $\int_C \dfrac{dz}{(z - z_0)} = 0$

(D) $\int_C \dfrac{dz}{(z - z_0)} = \dfrac{2\pi i}{3!}$

where n is any integer, except $n = 1$.

67. Let $G = \{0, 1, 2\}$ and define $*$ on G by $a * b = |a - b|$, $\forall\, a, b \in G$. The identity element of G is

(A) 0

(B) 1

(C) 2

(D) None of the above

68. If G is a finite group and H is a subgroup of G, then

(A) order of G divides order of H

(B) order of G equals order of H

(C) order of H divides order of G

(D) order of H does not divide order of G

69. Let G be a permutation group. Then an odd permutation is of

(A) odd order

(B) even order

(C) even or odd order

(D) None of the above

70. Let G be any group of order $2p$, where p is an odd prime. Then G is

(A) cyclic

(B) dihedral

(C) either cyclic or dihedral

(D) None of the above

71. Let G and G' be two groups and a homomorphism $f : G \to G'$ is one-one if and only if

(A) $\ker f = \phi$

(B) $\ker f \neq \phi$ and contains at least two elements

(C) $\ker f = \{e\}$

(D) None of the above

where e is the identity element.

72. In a permutation group G, consider the following statements :

P : The product of two even permutations is even.

Q : The product of two odd permutations is even.

Choose the correct answer.

(A) Both P and Q are true

(B) P is true but Q is false

(C) P is false but Q is true

(D) Both P and Q are false

73. The order of (8, 4, 10) in the group $Z_{12} \times Z_{60} \times Z_{24}$ is

(A) 60

(B) 24

(C) 12

(D) 6

The binary operations are usual.

74. Choose the correct statement.

(A) $\mathbb{Z}_2$ is an integral domain but not a field.

(B) $\mathbb{Z}_2$ is a field but not an integral domain.

(C) $\mathbb{Z}_2$ is both a field and an integral domain.

(D) None of the above

The binary operations are usual.

75. Let $G: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear mapping defined by

$$G(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$$

The dimension of image of G is

(A) 1

(B) 2

(C) 3

(D) None of the above

76. Consider the vector space $\mathbb{R}^3$ with usual vector addition and scalar multiplication. Let

$$U = \{(a, b, c) : a = b = c\}$$

Then

(A) U is a subspace of $\mathbb{R}^3$

(B) U is not a subspace of $\mathbb{R}^3$

(C) U is an empty set

(D) None of the above

77. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be a mapping defined as follows :

$$f(x, y) = (ax + by, cx + dy)$$

Then f is

(A) linear

(B) non-linear

(C) not bounded

(D) None of the above

78. Let W be a subspace of $\mathbb{R}^3$, such that $W = \{(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 0\}$. A basis of W will contain

(A) 3 elements

(B) 2 elements

(C) 1 element

(D) infinite elements

79. Any superset of linearly dependent vectors is

(A) basis

(B) linearly independent

(C) linearly dependent

(D) None of the above

80. Consider the following two statements :

P : An orthogonal set of non-zero vectors is linearly independent.

Q : An orthonormal set of vectors is linearly independent.

Choose the correct answer.

(A) Only P is true

(B) Only Q is true

(C) Both P and Q are false

(D) Both P and Q are true

81. A particle moves along a straight line according to the law $x^2 = 6t^2 + 4t + 3$, where x = distance, t = time. The acceleration varies as

(A) t

(B) $\frac{1}{x}$

(C) $\frac{1}{x^2}$

(D) $\frac{1}{x^3}$

82. A point moves in a straight line so that its distance x from a fixed point at time t is proportional to t^n. If v is the velocity and f is the acceleration at any time t, then

(A) $v = nfx$

(B) $f^2 = \frac{nvx}{n-1}$

(C) $v^2 = \frac{nfx}{n-1}$

(D) None of the above

83. A particle moves towards a centre of attraction starting from rest at a distance a from the centre; if its velocity when at any distance x from the centre varies as

$$\sqrt{\frac{a^2 - x^2}{x^2}}$$

Then the law of force is

(A) $\frac{d^2x}{dt^2} = \frac{-\mu^2 a^2}{x^3}$

(B) $\frac{d^2x}{dt^2} = \frac{-\mu^2 a^2}{x^2}$

(C) $\frac{d^2x}{dt^2} = \frac{-\mu a^2}{x^3}$

(D) $\frac{d^2x}{dt^2} = \frac{-\mu a^2}{x^2}$

where μ is a constant of proportionality.

84. A particle describes an ellipse under a force $\frac{\mu}{r^2}$ and has a velocity v at a distance r from the centre of force. Then the periodic time is

(A) $\frac{2\pi}{\sqrt{\mu}}\left(\frac{2}{r}-\frac{v^2}{\mu}\right)^{3/2}$

(B) $\frac{2\pi}{\sqrt{\mu}}\left(\frac{2}{r}-\frac{v^2}{\mu}\right)^{-3/2}$

(C) $\frac{2\pi}{\sqrt{\mu}}\left(\frac{2}{r}-\frac{v^2}{\mu}\right)^{1/2}$

(D) $\frac{2\pi}{\sqrt{\mu}}\left(\frac{2}{r}-\frac{v^2}{\mu}\right)^{-1/2}$

85. If a particle of mass m describes a circle of radius r with angular velocity ω, the only acceleration is towards the centre and is equal to

(A) $\omega^2 r$

(B) ωr^2

(C) ωr

(D) $\omega r^{1/2}$

86. The algebraic sum of the moments of any two forces about any point in their plane is

(A) equal to the moment of their resultant about the same point

(B) greater than the moment of their resultant about the same point

(C) less than the moment of their resultant about the same point

(D) None of the above

87. Three forces P, Q, R act along the sides BC, CA, AB of a triangle ABC, taken in order. If their resultant passes through the incentre of ΔABC, then

(A) $P+Q=R$

(B) $P+Q+R=0$

(C) $PQR=0$

(D) None of the above

88. The complete integral of

$$z = px + qy + \sqrt{p^2+q^2}, \; p = \frac{\partial z}{\partial x}, \; q = \frac{\partial z}{\partial y}$$

is

(A) $z = ax + by$

(B) $z = ax + by + \sqrt{a^2+b^2}$

(C) $z = ab$

(D) None of the above

Here a and b are arbitrary constants.

89. The solution of

$$xp + yq = z,\ p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y}$$

is

(A) $\phi(xy, xz) = 0$

(B) $\phi\left(\frac{x}{y}, \frac{x}{z}\right) = 0$

(C) $\phi\left(\frac{y}{x}, \frac{z}{x}\right) = 0$

(D) None of the above

where ϕ is any arbitrary function.

90. If $u + iv = \cot(x + iy)$, where x, y, u, v are real numbers, then

(A) $u^2 + v^2 - 2u\cot 2x = 0$

(B) $u^2 + v^2 + 2u\cot 2x = 1$

(C) $u^2 + v^2 - 2u\cot 2x = 1$

(D) None of the above

91. The solution of the linear programming problem

Min $Z = 200x + 500y$

subject to the constraints

$$x + 2y \geq 10$$
$$3x + 4y \leq 24$$
$$x \geq 0$$
$$y \geq 0$$

is

(A) $x = 3,\ y = 4$

(B) $x = 4,\ y = 3$

(C) $x = 2,\ y = 1$

(D) None of the above

92. The linear programming problem

Min $Z = 3x + 2y$

subject to the constraints

$$x + y \geq 8$$
$$3x + 5y \leq 15$$
$$x \geq 0$$
$$y \geq 0$$

has

(A) no solution

(B) more than one solution

(C) unique solution

(D) None of the above

93. A hyperplane is

(A) a convex set

(B) a concave set

(C) both convex and concave

(D) a basic feasible solution

94. Consider the following two statements :

P : The set of all feasible solutions (if non-empty) of a linear programming problem is a concave set.

Q : Intersection of any finite number of convex sets is also a convex set.

Choose the correct answer.

(A) Both P and Q are true

(B) Both P and Q are false

(C) P is false but Q is true

(D) P is true but Q is false

95. The remainder when 2^{340} is divided by 341 is

(A) -1

(B) 3412

(C) 9

(D) 1

96. The g.c.d. of -8 and 36 is

(A) 2

(B) -4

(C) 4

(D) None of the above

97. Imagine P as the largest known prime till date. Then choose the correct answer.

(A) $(P-1)! \equiv 1 \pmod{P}$

(B) $(P-1)! \equiv -1 \pmod{P}$

(C) $(P-1)! \equiv 4^P \pmod{P}$

(D) $(P-1)! \equiv (-4)^P \pmod{P}$

98. The remainder, when 19 divides

$$(1)^{19} + (2)^{19} + \cdots + (18)^{19}$$

is

(A) 0

(B) -1

(C) 1

(D) 19

99. The order of convergence of Newton-Raphson method is

(A) 1

(B) 1·5

(C) 2

(D) None of the above

100. By Newton-Raphson method, the obtained recurrence formula to find the square root of any number y is

(A) $x_{n+1} = \frac{1}{2}\left(x_n - \frac{y}{x_n}\right)$

(B) $x_{n+1} = \frac{1}{3}\left(x_n + \frac{y}{2x_n}\right)$

(C) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{y}{x_n}\right)$

(D) None of the above

Here x_n is the nth approximate of y.

SPACE FOR ROUGH WORK

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